

THE STRUCTURE SPACE AND IDEAL CENTRE OF A BANACH LATTICE

A. W. WICKSTEAD

BANACH LATTICES

A Banach lattice is a Banach space E equipped with a partial order " $\geq$ " with the properties:

$$(1) \quad x \geq y \Rightarrow x + z \geq y + z \quad (x, y, z \in E)$$

$$(2) \quad x \geq y, \lambda \in \mathbb{R}_+ \Rightarrow \lambda x \geq \lambda y \quad (x, y \in E)$$

(3) Any pair $x, y \in E$ has a least upper bound $x \vee y$ and a greatest lower bound $x \wedge y$.

$$(4) \quad x, y \in E \text{ and } |x| \leq |y| \text{ together imply } \|x\| \leq \|y\| \quad (\text{where } |x| = x \vee (-x)).$$

The order is completely determined by the set $E_+ = \{x \in E : x \geq 0\}$ as $x \geq y$ if and only if $x - y \in E_+$.

Examples (1) If Σ is a locally compact Hausdorff space and $C_0(\Sigma)$ the space of all continuous real valued functions on Σ which "vanish at infinity" then $C_0(\Sigma)$ is a Banach space for the supremum norm. If we define $x \geq y$ to mean $x(\sigma) \geq y(\sigma)$ for all $\sigma \in \Sigma$ then $C_0(\Sigma)$ becomes a Banach lattice.

(2) If μ is a Radon measure on the locally compact Hausdorff space S the Banach spaces $L^p(\mu)$ ($1 \leq p \leq \infty$) can be made into Banach lattices by defining $x \geq y$ to mean $x(s) \geq y(s)$ except on a locally μ -null set.

A linear subspace J of a Banach lattice E is an ideal if $x \in J, y \in E$ and $|y| \leq |x|$ imply $y \in J$. With the inherited norm and order a closed ideal is a Banach lattice. So also is the quotient E/J when given the cone $p(E_+)$, where $p : E \rightarrow E/J$ is the canonical quotient map. E.g. if $E = C_0(\Sigma)$ and K is a closed subset of Σ then $J = \{f \in E : f|_K = 0\}$ is a closed ideal and E/J may be identified with $C_0(K)$.

A topological order unit (t.o.u.) for a Banach lattice E is an element $e \in E_+$ such that the smallest closed ideal in E containing e is the whole of E . E.g. if $E = L^p(\mu)$ ($1 \leq p \leq \infty$) then E has a t.o.u. if and only if Σ is σ -finite, and any function which is strictly positive almost everywhere is a t.o.u.

If $E = C_0(\Sigma)$ then E has a t.o.u. if and only if Σ is σ -compact, and any strictly positive function is a t.o.u. If E has a t.o.u. and J is a closed ideal in E then E/J has a t.o.u. but J need not have.

Banach lattices with t.o.u.'s are very nice to handle, mainly because of the representation theorem of Davies [8] and Lotz (see [15], Theorem III.4.5). In general Banach lattices need have no maximal ideals. For many of our purposes it suffices to suppose the ^{existence of} many closed ideals for which the quotient has a t.o.u. To be precise we call E semi-simple if for each $e \in E_+ \setminus \{0\}$ there is a closed ideal J such that E/J has a t.o.u. and $p(e) \neq 0$, where $p : E \rightarrow E/J$ is the canonical projection. The author knows of no example of a Banach lattice which is not semi-simple. An ideal in a semi-simple Banach lattice is semi-simple. If all quotients of a Banach lattice are semi-simple we term it simple.

A Banach lattice E is an AL-space if $x, y \in E_+$ implies $||x+y|| = ||x|| + ||y||$, and is an AM-space if $x, y \in E_+$ implies $||x \vee y|| = ||x|| \vee ||y||$. AL-space, AM-spaces and Banach lattices with a t.o.u. are all simple.

The Banach dual of the Banach lattice E , E^* , ordered by $f \geq g$ if and only if $f(x) \geq g(x)$ ($x \in E_+$) is also a Banach lattice.

We say the net (x_γ) is directed downward to 0 if $\gamma \geq \gamma' \Rightarrow x_\gamma \leq x_{\gamma'}$, and $\inf \{x_\gamma\} = 0$. $E^\times$ is the subspace of linear functionals f in E^* such that $f(x_\gamma) \rightarrow 0$ whenever (x_γ) is directed downward to 0. The norm in E is order continuous if $||x_\gamma|| \rightarrow 0$ whenever (x_γ) is directed downward to 0; this is equivalent to $E^* = E^\times$.

A Banach lattice E is Dedekind (σ -) complete if every (countable) set $A \subseteq E$ which has an upper bound has a least upper bound. The spaces $L^p(\mu)$ are Dedekind complete, $C_0(\Sigma)$ is Dedekind complete if and only if Σ is extremally disconnected.

THE IDEAL CENTRE

The ideal centre of a Banach lattice E , $Z(E)$, consists of all linear operators T on E for which there is $\lambda \geq 0$ with $-\lambda x \leq Tx \leq \lambda x$ for all $x \in E_+$. It is easy to check that $Z(E)$ is an algebra of bounded linear operators on E . $Z(E)$ may be ordered by defining $S \geq T$ to mean $Sx \geq Tx$ for all $x \in E_+$. $Z(E)$ certainly contains the band projections, i.e. those linear operators P on E with $0 \leq P = P^2 \leq I$, but is generally much larger than the closed algebra they generate.

There is a compact Hausdorff space Ω and an isometric algebra isomorphism of $Z(E)$ with the operator norm onto $C(\Omega)$, the space of all continuous real valued functions on Ω , with the supremum norm. In particular $Z(E)$ is abelian and a lattice.

Examples. (1) The elements of $Z(C_0(\Sigma))$ are precisely the operators of the form $f \rightarrow f \cdot \phi$ where ϕ is a fixed bounded continuous function on Σ .

(2) $Z(L^p(\mu))$ is the algebra of multiplication pointwise locally μ -almost everywhere by $L^\infty(\mu)$ functions.

In many ways the structure of $Z(E)$ is simpler than that of E itself, and it also has the added structure of an algebra. In spite of the simplification $Z(E)$ does sometimes retain a lot of information about E . For example if E has a t.o.u. then E is Dedekind (σ -) complete if and only if $Z(E)$ is.

For general Archimedean vector lattices some rather larger algebras of operators have been studied. $S(E)$ is the set of all linear operators T on E for which for each $x \in E_+$ there is $\lambda_x \geq 0$ with $-\lambda_x x \leq Tx \leq \lambda_x x$. A positive orthomorphism is a positive linear operator on E such that $x \wedge y = 0$ implies $Tx \wedge y = 0$ ($x, y \in E_+$). $P(E)$ is the space of all differences of positive orthomorphisms on E . Both $S(E)$ and $P(E)$ are abelian lattice

ordered algebras. Clearly $Z(E) \subseteq S(E) \subseteq P(E)$ and in general both inclusions can be proper. $P(E)$ and $S(E)$ have been studied in [4], [5], [7], [21] and [23].

Proposition [21]. If E is a Banach lattice then $Z(E) = S(E) = P(E)$.

Another characterisation of $Z(E)$ that is of use in the proof of later results is:

Proposition. ([19], Theorem 5.2). A linear operator T on E lies in $Z(E)$ if and only if it is norm bounded and leaves every closed ideal in E invariant.

REPRESENTING THE IDEAL CENTRE

We know that $Z(E)$ is isomorphic to some space of functions, but as yet we have no concrete space on which these functions are defined nor any idea of how the elements of $Z(E)$ act on E .

Suppose E is an AM-space then there are sufficiently many one dimensional ideals in E^* that their weak* closed linear span is the whole of E^* . We let $\Delta(E^*)$ denote the set of all one-dimensional ideals in E^* . $\Delta(E^*)$ may be topologised by taking for the closed sets in $\Delta(E^*)$ the sets of the form $\{L \in \Delta(E^*) : L \subseteq F\}$ for F varying over the weak* closed ideals in E^* . Wils ([22], Theorem 2.8) has proved that $Z(E)$ may be identified with the space of bounded real valued functions on $\Delta(E^*)$ which are continuous for this topology, $C^b(\Delta(E^*))$. The correspondence is such that if $T \in Z(E)$ corresponds to $\phi_T \in C^b(\Delta(E^*))$ then

$$f(Tx) = \phi_T(f).f(x)$$

whenever $x \in E$ and f lies in a one-dimensional ideal in E^* .

In general E^* need have no one-dimensional ideals so there is no hope of generalising this result as it stands. A one-dimensional ideal in E^* determines the family of all weak* closed ideals of E^*

which contain it. This family is maximal with respect to the property that finite intersections of its members are non-zero. Even for AM-space there will, in general, be other such families not determined by one-dimensional ideals. We use these families as a substitute for the missing one-dimensional ideals.

Definition. The structure space of the Banach lattice E , $M(E)$, is the set of all families F of weak* closed ideals in E^* which are maximal with respect to the property that finite intersections of members of F are not $\{0\}$. $M(E)$ is topologised by taking as a base for the closed sets the sets $G^V = \{F \in M(E) : G \in F\}$ where G ranges over the weak* closed ideals of E^* .

$M(E)$ is a compact T_1 space. For $C_0(\Sigma)$ there is a natural correspondence between weak* closed ideals in $C_0(\Sigma)^*$ and closed subsets of Σ , so $M(C_0(\Sigma))$ is homeomorphic to the Wallman compactification of Σ ([13], Problem 5R). This is Hausdorff if and only if Σ is normal, so $M(E)$ is not always Hausdorff. If E has a t.o.u. there is an alternative description of $M(E)$ which shows that $M(E)$ is Hausdorff in that case (see [8]). If J is a closed ideal in E and $M(E)$ is Hausdorff then $M(E/J)$ is Hausdorff, but $M(J)$ need not be.

If $L \in \Delta(E^*)$ then $\rho(L) = \{F : F \text{ is a weak* closed ideal in } E^*, L \subseteq F\} \in M(E)$. It is easily verified that ρ is a homeomorphism of $\Delta(E^*)$ into $M(E)$.

Proposition. ([19], Proposition 4.1) For a Banach lattice E the following are equivalent:

- (1) $\rho(\Delta(E^*)) = M(E)$.
- (2) E^* is the ^{weak*}closed linear span of its one-dimensional ideals
and $\Delta(E^*)$ is compact.
- (3) E is homeomorphically order isomorphic to $C(M(E))$.

The structure space gives us a useful representation of $Z(E)$, at least when E is semi-simple.

Theorem. ([19], Theorem 4.4; [9], Corollary 3.5) Let E be a semi-simple Banach lattice. There is an isometric order and algebra isomorphism $T \rightarrow T^\pi$ of $Z(E)$ onto $C(M(E))$ such that for F a weak* closed ideal in E^* and $T \in Z(E)$, $T^*|_F \geq 0$ if and only if $T^\pi|_{F^\vee} \geq 0$.

The reasons for studying both the structure space and ideal centre of a Banach lattice at the same time are:

- (a) The methods used in proofs are often similar.
- (b) The proofs of results for the ideal centre and structure space are often interdependent.
- (c) Some results about the ideal centre actually require conditions on the structure space.

THE SIZE OF THE IDEAL CENTRE

If we are to obtain information about a Banach lattice through the study of its ideal centre then the ideal centre will have to be fairly large. In what sense can it be said to be large and when can we guarantee that this occurs?

Definition. If X is any Banach space and A an algebra of bounded linear operators on X then $x \in X$ is a cyclic vector for A if $\{Tx : T \in A\}$ is dense in X .

Proposition. If E is a Banach lattice there is a cyclic vector for $Z(E)$ if and only if E has a t.o.u.

It is of interest to note that algebras of operators isometric to $C(\Omega)$ with cyclic vectors can only arise in this way:

Theorem. ([18]). Let X be a real Banach space and A an algebra of bounded linear operators on X which is isometrically algebra isomorphic to some space $C(\Omega)$. Then X can be given an order for which it is a Banach lattice and such that $A \subseteq Z(X)$.

Alternative conditions for $Z(E)$ to be large are contained in:

Theorem. If E is a Banach lattice satisfying (1) or (2) then $Z(E)$ is a maximal Abelian algebra of bounded linear operators on E .

(1) There is a simple Banach lattice E_0 with Hausdorff structure space such that E is an ideal in E_0 .

(2) E is Dedekind σ -complete.

For (1) this is Corollary 3.3 of [20] and for (2) is unpublished.

Gouillet de Rugy has given an example ([10], Proposition 3.42) of an AM-space G such that the only real valued continuous functions on $\Delta(G^*)$ are the constants. It follows from Wils' description of $Z(G)$ that it contains only multiples of the identity on G . Clearly in such cases we can expect to learn nothing of G from $Z(G)$.

IDEALS, QUOTIENTS AND DUALS

Three important ways of creating new Banach lattices from an old one are by forming closed ideals, quotients by those ideals and dual spaces. Clearly we would like to know what happens to the structure space and ideal centre after these operations.

Let J be a closed ideal in the Banach lattice E . If $T \in Z(E)$ then $TJ \subseteq J$. It is easily checked that $T|_J \in Z(J)$. We can define T_J on E/J by $T_J(x + J) = Tx + J$, then $T_J \in Z(E/J)$.

Proposition. (Unpublished) Let J be a closed ideal in the Banach lattice E .

- (1) If E is Dedekind complete $\{T|_J : T \in Z(E)\} = Z(J)$.
- (2) If E has a t.o.u. then $\{T|_J : T \in Z(E)\}$ is dense in $Z(J)$ for the strong operator topology.

The density result is not true in general. Take G to be the M -space of Goullet de Rugy and J any closed ideal of G generated by a single element of G_+ which is not one-dimensional.

Proposition. ([19], Proposition 5.1) If E is a semi-simple Banach lattice with Hausdorff structure space and J a closed ideal in E then $\{T|_J : T \in Z(E)\} = Z(E/J)$.

The conclusion fails if the structure space is not Hausdorff, as can be seen by choosing J a closed ideal in G such that $2 \leq \dim(G/J) < \infty$.

If $T \in Z(E)$ then its adjoint T^* lies in $Z(E^*)$.

Theorem. ([17], Theorem 5.2). The Banach lattice E has an order continuous norm if and only if $\{T^* : T \in Z(E)\} = Z(E^*)$.

When we come to deal with the structure space quotients behave much more nicely than ideals.

Proposition. ([19]) If J is a closed ideal in the Banach lattice E , $M(E/J)$ is homeomorphic to the subset $(J^0)^V$ of $M(E)$.

Theorem. ([20], Corollary 3.5) Let E be a semi-simple Banach lattice, J a closed ideal in E and both E and J have Hausdorff structure space. Then $M(J)$ is homeomorphic to the Stone-Cech

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compactification of $M(E) \setminus (J^0)^V$.

STRONG OPERATOR TOPOLOGY ON $Z(E)$ - GENERAL PROPERTIES

The strong operator topology on $Z(E)$ is the locally convex topology generated by the semi-norms $T \mapsto ||Tx||$ for $x \in E$. The weak operator topology is that generated by the semi-norms $T \mapsto |f(Tx)|$ for $x \in E$, $f \in E^*$. We let $Z(E)_s$ (respectively $Z(E)_w$) denote $Z(E)$ with the strong (respectively weak) operator topology.

THEOREM. ([17], Theorem 2.1) $Z(E)_s$ is a topologically complete locally convex lattice.

If X_T is a locally convex space the strong topology on $(X_T)'$, the topological dual of X_T , is the topology of uniform convergence on bounded subsets of X_T . $(X_T)'_B$ denotes $(X_T)'$ with this topology. Recall also that X_T is semi-reflexive if the evaluation map of X_T into $((X_T)'_B)'_B$ is onto, and is reflexive if it is also a homeomorphism.

Proposition. ([17]) The completion of $(Z(E)_s)'_B$ can be given a norm which defines its topology and for which it is an AL-space. $(Z(E)_s)'$ is an ideal in this completion.

STRONG OPERATOR TOPOLOGY ON $Z(E)$ - USE IN CHARACTERISATIONS

In characterising various classes of Banach lattices by means of the ideal centre we must assume the ideal centre is large. The easiest assumption to make is that of the existence of a t.o.u.

Proposition. ([17], Proposition 2.2) If E is a Banach lattice with a t.o.u. the following are equivalent:

- (1) E is homeomorphically order isomorphic to a space $C(\Omega)$.
- (2) $Z(E)_s$ is normable.

- (3) $Z(E)_s$ is metrizable.
- (4) $Z(E)_s$ is barrelled.
- (5) $Z(E)_s$ is of the second category in itself.

Proposition. ([17]) If E is a Banach lattice with a t.o.u. the following are equivalent:

- (1) E is finite dimensional.
- (2) $Z(E)_s$ is reflexive.

Theorem. If E is a Banach lattice with a t.o.u. the following are equivalent:

- (1) E has an order continuous norm.
- (2) Every order interval in E is $\sigma(E, E^*)$ compact.
- (3) If the net (T_γ) in $Z(E)$ is directed downward to 0 then $T_\gamma \rightarrow 0$ for the strong operator topology.
- (4) $Z(E)_s' \subseteq Z(E)^X$.
- (5) $Z(E)_s$ is semi-reflexive.
- (6) Every order interval in $Z(E)$ is $\sigma(Z(E), Z(E)_s')$ compact.
- (7) Every order interval in $Z(E)$ is compact for the weak operator topology.
- (8) $Z(E)_w$ is sequentially complete.

The equivalence of (1) to (7) is Theorem 5.1 of [17], whilst that of (8) has not been published.

Proposition. (Unpublished) If the Banach lattice E either has a t.o.u. or is Dedekind complete then the following are equivalent:

- (1) Every order interval in E is norm compact.
- (2) Every order interval in $Z(E)$ is compact for the strong operator topology.

SPECIAL SUBSETS OF $Z(E)$

Two subsets of $Z(E)$ are of special interest, namely the compact operators in $Z(E)$ and the isometries.

Theorem. ([17], Theorem 4.1) If E is any Banach lattice and $T \in Z(E)$ the following are equivalent:

- (1) T is compact.
- (2) There is $f \in c_0(\Delta(E))$ such that for $y \in E$

$Ty = \sum_{L \in \Delta(E)} f(L) p_L(y)$, where p_L is the band projection onto L .

Furthermore $\|T\|$ is the same as the supremum norm of f .

Proposition. ([16], Theorem 5 and [17], Proposition 4.2) For any Banach lattice E the following are equivalent:

- (1) The compact operators in $Z(E)$ are dense in $Z(E)$.
- (2) E is the closed linear span of its one dimensional ideals.
- (3) Every order interval in E is norm compact.

Let $I(E)$ denote the set of isometries in $Z(E)$. For $T \in Z(E)$ it is easily checked that $T \in I(E)$ if and only if $|T| = I$. It is also clear that $I(E)$ is a topological group for the strong operator topology.

Theorem. (Unpublished) If E is a Dedekind complete Banach lattice the following are equivalent:

- (1) $I(E)$ is compact for the strong operator topology.
- (2) Every order interval in E is norm compact.
- (3) The linear span of the common eigenvectors of $I(E)$ is

dense in E .

A CATEGORICAL APPROACH

Nowadays, whenever we have a natural association of one object with another, an algebraist is likely to ask if the association can be made functorial. This is possible both for the ideal centre and the structure space provided we work on appropriate categories.

The morphisms allowed from the Banach lattice E into the Banach lattice F will be those lattice homomorphisms $T : E \rightarrow F$ such that the ideal generated by TE is dense in F . Note that it is possible that $\text{Hom}(E, F) = \emptyset$. $\mathcal{B}$ will denote the category of all Banach lattices with these morphisms, and $\mathcal{B}_0$ the full subcategory whose objects are semi-simple Banach lattices with Hausdorff structure space.

$\mathcal{C}$ will denote the category of vector lattices with distinguished order units and order unit preserving lattice homomorphisms between them. $\mathcal{T}$ will denote the category of compact Hausdorff spaces and continuous maps between them.

Theorem. ([17], Theorem 3.1) There is a unique covariant functor Z from $\mathcal{B}$ to $\mathcal{C}$ which assigns to each object E in $\mathcal{B}$ the pair $(Z(E), I_E)$ and to each morphism $T : E \rightarrow F$ a vector lattice homomorphism $Z(T) : Z(E) \rightarrow Z(F)$ with $Z(T)I_E = I_F$ and such that $(Z(T)S) \circ T = T \circ S$ for all $S \in Z(E)$.

The strong operator topology appears to have little of a role to play here. Examples can be given to show that $Z(T)$ need not be continuous, open or closed. It must be continuous on bounded sets and have a closed graph, but these properties seem of little use.

For Banach lattices with topological order units $Z(T)$ does reflect some properties of T :

Theorem. ([17], Theorem 3.2) Let E and F be Banach lattices with topological order units e and f respectively and $T : E \rightarrow F$ a lattice homomorphism with $Te = f$.

- (1) TE is dense in F if and only if $Z(T)Z(E)$ is dense in $Z(F)$ for the strong operator topology.
- (2) TE is an ideal in F if and only if $Z(T)Z(E) = Z(F)$.
- (3) T is $(1,1)$ if and only if $Z(T)$ is $(1,1)$.

Theorem. ([20], Corollary 2.2) There is a unique contravariant functor M from $\mathcal{B}_0$ to $\mathcal{T}$ which assigns to each object E in $\mathcal{B}_0$ its structure space and to each morphism $T : E \rightarrow F$ a continuous map $M(T) : M(F) \rightarrow M(E)$ such that, for each weak* closed ideal G of E^* , $M(T)(T^{*-1}G)^{\vee} \subseteq G^{\vee}$.

Theorem. Let E and F be objects in $\mathcal{B}_0$ and $T : E \rightarrow F$ a morphism.

- (1) $M(T)$ is onto if and only if T is $(1,1)$.
- (2) If E is simple then $M(T)$ is $(1,1)$ if and only if TE is an ideal in F .

[20], Proposition 2.3, the note thereafter and Proposition 2.4.

OPEN PROBLEMS

1. Are there any Banach lattices which are not semi-simple?
2. Do Dedekind complete Banach lattices have Hausdorff structure space?
3. If J is a closed ideal in the Banach lattice E and both $M(J)$ and $M(E/J)$ are Hausdorff must $M(E)$ be Hausdorff?
4. If $Z(E)$ is a maximal Abelian algebra of bounded linear operators on E is it true that E is $(\sigma-)$ complete if and only if $Z(E)$ is?
5. Is there a natural map of $M(E^*)$ onto $M(E)$?
6. Describe $Z(E)_{\mathcal{B}_0}$. In particular are all its elements of the form $T \mapsto f(Tx)$ ($x \in E, f \in E^*$)?
7. If E and F have t.o.u.'s and $Z(E)_{\mathcal{B}_0}$ and $Z(F)_{\mathcal{B}_0}$ are homeomorphically

order isomorphic must the same be true of E and F ?

8. Is it always true that $I(E)$ is compact if and only if the linear span of the common eigenvectors of $I(E)$ is dense in E ?

9. Is $Z(T)$ always continuous for the weak operator topologies?

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Department of Pure Mathematics,
The Queen's University of Belfast,
Belfast BT7 1NN.
Northern Ireland.