Using MATHEMATICA with mathematics students

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Rationale

For many years I have been developing the use of computers in teaching topics in the traditional university mathematics curriculum. For almost as long I have been increasingly relying on computers in my own research. A use of computers that, until 1989, I had no experience of was Computer Algebra, the use of computers to carry out symbolic manipulations, which may be in analysis as much as in algebra! What I had read suggested that this was a use of computers which would become increasingly important for non-academic as well as academic mathematicians and was likely, in the long term, to have a significant effect on the content of the undergraduate curriculum. As so many of us do, I decided to learn about this topic by teaching a course on it. In the 1989/90 academic year I offered an option on Computer Algebra in an M.Sc. course on Computational Science using an, at the time rather dated, program called REDUCE. In spite of significant problems with running this software on a central mini-computer and its lack of many features, such as graphics, that are now taken for granted in computer algebra software, the course was enough of a success that in 1990/91 I started a final year undergraduate course on computer algebra.

There were obviously going to be a number of changes needed in the transition from teaching a short course for postgraduate scientists to teaching a full term course for undergraduate mathematicians. One thing that was very obvious after my trial year was that an important part of the course would be the use of mathematical knowledge to check on the results produced by the software. Every Computer Algebra program that I have used can be set a problem to which it gives answers that are erroneous in some manner and I regarded the development of critical use of the results from a computation as being an important part of the objectives of such a course. The lack of graphics clearly needed to be remedied. Apart from its use as an attention grabber, no-one actually using computer algebra software uses it without some graphical capability, even if it is provided by an external program. Use of a program with integrated graphics was clearly desirable and by now all the more popular computer algebra programs have this. A final desire in my design of the course was to teach a wide range of techniques that could be performed by a computer algebra package without requiring a substantial mathematical knowledge, although I was prepared to ask for quite a lot of mathematical maturity. I hoped that course would attract students from a wide variety of different mathematical backgrounds and it has indeed done so.

Description

I settled on using, what was at the time a relatively new program, MATHEMATICA on PC's. The decision as to the choice of program was made on advice that users of it gave me at the time, whilst the hardware platform was dictated by university policy at the time. If forced to make such a decision at this time I would not necessarily make the same one, but our experience with this configuration has been sufficiently satisfactory that I might well do. The mode of use of the software that we have adopted means that students only need access to a printer at the end of a session with the machine so that sharing of printers is feasible.

The course starts by demonstrating some of the capabilities of MATHEMATICA and then shows them how to start the program, edit their input, refer to past input and output, save their work and how to get printed copy of their session on the machine. One lecture of this has been adequate to allow them to use the software without any further assistance.

They are then told about numbers, which can be of many different kinds in computer algebra software, which brings home immediately some of the important differences between computer algebra programs and most numerical software. The important topic of pattern matching is introduced early for the same reason and also because it is important for a smooth development of the rest of the course. They are then told how to manipulate symbolic expressions of various kinds. As a bit of light relief, we then look at graphics - mainly two dimensional because three dimensional graphics are too slow to compute. Differential and integral calculus follow this and then equation solving. As an introduction to the programmability of MATHEMATICA, students are then introduced to its various looping constructions which are illustrated by, for example, a repeated bisection root-finder. Vectors and matrices come next, quite late in the course, because teaching it in a straightforward way involves references to many other constructions which have taken until now to appear. Approximately the final 30% of the course is taken up by case studies illustrating and reinforcing many of the topics that have been introduced previously. The topics are currently taken from calculus, algebra, and number theory, but are updated each year.

Students are lectured to, with computer demonstrations, for three hours a week. They are given a set of problems to attempt for themselves which are then marked, returned to them, and discussed at a tutorial. The problems are initially quite short, but by the end of the course a smaller number of more substantial problems are posed.

The course is examined by a single three hour practical examination, into which students are allowed to take all the notes that are provided for the course and any other notes of their own. They are given a number of short problems to attempt all of, plus some longer problems that offer them some choice. For examination purposes, all interaction between students and program are automatically logged for the examiner's benefit - students do not have to decide which parts of their session were important, I just disregard false starts.

Evaluation

There were several practical problems encountered. We have already had to increase our provision of computers, printers and software twice because of increased demand for the course. This increased demand has also involved us in an ever increasing amount of marking work. Programs such as MATHEMATICA are upgraded regularly in significant ways, which involves not only changes to our equipment but also to the course - substantial annual rewriting of the course has been necessary. We have also encountered substantial logistic difficulties in the examining process. This has involved, for example, holding two examination sessions abutting each other in time and ensuring that no contact between the two groups was possible. All of these problems have however been overcome without undue difficulty.

The students have reacted favourably to the course. Although only about three hours of practical work a week seem to be necessary for successful completion of their assignments, many students spend considerable time in the evening, at weekends and over vacations in extra work. This does not seem to have affected their performance in other courses as the examination of this course is undertaken well before our main summer examination period gets underway (an extra attraction of this course for some students).

One unexpected spin-off of this course has been that several members of staff have attended the course and begun to use MATHEMATICA in their research, sometimes in areas that were rather unexpected such as the psychology of three dimensional vision!

The course has, from the start, attracted mathematics students not only from our own university but also from two local teacher training colleges. In the 1992/93 session it was also made available to students of computer science here and immediately became moderately popular with them, much more so than we would have expected any course in mathematics to be. During the first three years that the course has been taught the numbers taking it have grown from 24, to 42 and then to 55. Initial indications are that this years class will be even bigger and we have already organised a bigger, but less convenient, lecture theatre for the class which will necessitate a portable computer for lecture demonstrations. Presumably at some stage we will have to put a limit on the numbers taking the course, but we have no desire to prevent students taking a useful course that they enjoy.