estimates for solutions, including regularity near the boundary, are proved in detail. The book is essentially self-contained.

In the process of solving particular problems, many delicate methods for obtaining *a priori* estimates for partial differential equations are illustrated. These include choosing appropriate test functions to multiply the two sides of the equation, appropriate change of variables, finite difference methods, eigen-expansions for hyperbolic equations, and complex analysis estimates for Schrödinger equations. More systematic study is devoted to Aleksandrov and Bony maximum principles and density theorems in Chapters VII and VIII.

As the author writes in the introduction, the methodology adopted in this book is but one of many approaches to non-linear problems. The strong point of this book, I think, is that it concentrates on only one approach and has developed it well. After reading this book, the reader is expected to understand the flavour of the approach, to have gained a lot of knowledge on how to make non-trivial *a priori* estimates, and also to be aware of the restrictions on growth conditions to the equations that this approach requires. Many of the complicated computations are treated here with great care.

This book serves as a good introduction for postgraduate students at the beginning of their study, who want to be familiar with one very important aspect of how to study non-linear partial differential equations. For advanced researchers, it is also a good reference to many useful results and to various kinds of techniques used in obtaining *a priori* estimates. Some of them are very subtle indeed, and can be helpful in solving other problems.

QI TANG

BANACH LATTICES

By P. MEYER-NIEBERG: 395 pp., DM.78.–, ISBN 3 540 54201 9 (Springer, 1991).

All the classical Banach spaces, such as $C(\Omega)$ and $L^p(\mu)$, are Banach lattices, so the lack of general interest in, and awareness of, Banach lattices amongst analysts is surprising. This is even more so when one realises that most applications of Banach spaces outside mathematics are in contexts where elements of the Banach space represent something which is being measured, and only elements which are 'positive' in some sense correspond to physically meaningful situations. Similarly, the operators which represent transitions from one state to another will, if they correspond to reality, take positive elements into positive ones. All I have said so far suggests only that we look at ordered Banach spaces and positive operators on them, but the numerous situations in which Banach lattices do form the natural space on which to model some phenomenon, plus the abundance of results that are valid for Banach lattices, but not more generally, make the study of Banach lattices worthwhile. There are many examples of applications of Banach lattices given in Aliprantis's review of this book in the *Bulletin of the American Mathematical Society*. That review also contains an extensive bibliography of other works on this and closely related topics.

Relatively recently, those working on the theory of abstract Banach spaces have discovered that those Banach spaces which can be given a Banach lattice structure form a much more nicely behaved class than the class of all Banach spaces. Section

5.4 of the book under review, on the 'Radon–Nikodym property in Banach lattices', beautifully exemplifies how much better behaved Banach lattices are than Banach spaces in general.

Research into what might be termed 'pure' Banach lattice (and Riesz space) theory has proceeded virtually independently for the last half century in Japan, the former Soviet Union and Western Europe, and it is only recently that the work done in each of these 'schools' is becoming known to the others. Even in Western Europe there are distinct groups originating from Germany and the Netherlands, with most (but not all) workers in other countries being descended from one of the four schools mentioned. Nowadays, these schools are at least aware of each other's existence, and refer to and build on each other's work, whilst the relatively recent interest in Banach lattices by Banach space workers has proceeded virtually independently of other work on Banach lattices. One nice feature of this book is that it contains material from both these traditions, and neatly integrates it.

The content of this book is mainly fairly predictable, although there is much recent material in it which has not previously found its way into books. The first chapter deals with the theory of Riesz spaces in general, as far as it is needed for the study of Banach lattices, including the fundamental results about linear operators on Riesz spaces. The second chapter deals with the theory of the classical Banach lattices, including the classical representation theorems for abstract M-spaces and L^p-spaces. It also contains some fairly technical material, which is needed for both this and subsequent chapters. The third and fourth chapters deal with operators on Banach lattices, with the fourth being devoted to their spectral theory. The final chapter is entitled 'Structures in Banach lattices', but is really about the ways in which Banach lattices are special as Banach spaces. In spite of the different historical origin of much of the material in this chapter, the author has succeeded in knitting it into the book in a seamless manner. To the reviewer's mind, this last chapter is the one that makes this book stand out from the others that already exist.

This book is by no means perfect. In common with many other books in this field, more examples would not come amiss. For example, after proving that order bounded operators into a Dedekind complete Riesz space are regular, the reader is referred to [1] for an otherwise unpublished example of Lotz, to see that the condition of Dedekind completeness cannot be dropped. Even if there were no room for such an example in this book, surely a reference to a published example, such as Kaplan's original example in [3], would have been possible. Similarly, in Section 3.1 there are a number of conditions given, each of which ensures that the centre of a Banach lattice is quite large. Even if it could not be included, Goullet de Rugy's example in [2], of an M-space with trivial centre, should have been referred to.

Aliprantis's review has referred to the lack of credit given to non-German mathematicians in this book, and Abramovich in his review in *Mathematical Reviews* has given many examples of this, so I shall not expand on it here.

The book was printed from camera-ready copy supplied by the author, and contains a distractingly large number of misprints, mainly minor, most of which could have been spotted by the use of a spelling checker. The not infrequent mistakes in names of mathematicians are particularly unfortunate. With the increasing number of books being produced in this way, it may be timely to remind potential authors that the normal sub-editing process will not be available to correct typographical errors, and that more care than usual is needed in production of camera-ready copy.

In summary, though, the book is a very worthwhile addition to the literature on

the subject, especially because of its final chapter. It is suitable either as a reference book or as an advanced undergraduate or postgraduate text, containing many exercises appropriate for the latter use. It is thoroughly to be recommended to those interested either in Banach lattices and their applications or in abstract Banach space theory.

References

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A. W. WICKSTEAD

DISCRETE LINEAR CONTROL SYSTEMS

By V. M. FOMIN: 284 pp., US$134.00, ISBN 0 792 31248 1 (Kluwer Academic Publishers, 1991).

The style of a book is a matter of taste. This book follows tradition in many respects, giving a rigorous, concise account of a field concentrating on the most important theoretical results, peppered with the individual interests of the author. The style was certainly to my liking. It might not be to everyone's liking, as some might need more examples and illustrations to make the new concepts familiar.

A remarkable feature of the book is that systems with both unstructured and stochastic disturbances are discussed and that the author was able to fit in most of the important theoretical results in an impressively short 284 pages. Considering the large scope of the theory that the book covers, it is forgivable that issues of practical implementations are rarely mentioned. There are no applications; the book is a collection of crystallized theory in elegant formulations, a potentially perfect territory of joy for a mathematician interested in the field. Practical control engineers probably would not think of it as easy reading with rules of thumb for their everyday jobs.

The proofs of theorems are in appendices after each chapter, which increases clarity and makes it easier to grasp the essential points of the topics discussed.

Chapter 1 introduces the basic concepts of control theory. The control object, the control algorithm and objective are defined in a general form, the control objective and strategies are classified, and optimal control is introduced.

Chapter 2 is devoted to finite-time period control, where dynamic programming, Bellman's equation, is considered first in general and then in special cases of linear quadratic-cost deterministic and stochastic systems. The Bayesian control strategy is discussed next under regularity conditions, and the solution of the linear quadratic Gaussian (LQG) problem is derived.

Infinite-time period control is the topic of Chapter 3. Ljapunov's stabilization method is discussed with examples. For linear systems the optimization problem is reduced to the solution of the matrix Riccati equation, with Lur'e's equation in the stationary case. Optimal linear regulators are also derived through the transfer function method, the polynomial approach. The minimax control problem is stated and solved under the assumption of bounded additive noise.

One of the most interesting chapters is Chapter 4, on control systems with